RESEARCH ON THE APPLICATION OF STATISTICAL ENERGY ANALYSIS TO THE BRIDGE-BORNE NOISE

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ABSTRACT. The purpose of this paper is to determine a method capable to predict noise both low and high frequency. Another purpose is to determine a boundary between Statistical Energy Analysis and Finite Element Analysis for noise prediction. An analytical model of a Z-shaped plate structure is used, and a set of power flow equation is formulated for the plate structure in order to perform SEA. To solve these equations, SEA parameters such as CLF, ILF, and mode count are estimated. A FEM model of the structure is also made to perform Modal Analysis, First 20 modes are evaluated, and Harmonic Analysis. An experiment measuring sound pressure level for each plate is also carried out to confirm the analytical results. The results for this type of structure, confirmed that SEA is valid above 63Hz and FEM is valid below 31.5Hz in octave band. Moreover, a new concept of evaluating this boundary between SEA and FEM using mode count is also proposed.

Keywords: Statistical Energy Analysis, Finite Element Analysis, Plate Structure, Bridge-borne noise, Sound Pressure Level, Mode count, ILF, CLF.

1. Introduction

The noise pollution in residential areas is an important issue for automotive and railway transport over steel bridges. There are several sources of that noise: the wheel/rail interaction, the engine noise, aerodynamics noise, and the noise caused by vibrations of steel bridges, rail track for railway bridges, or their support. The later is mainly recognized on bridges because the vibrations excite the bridge structure, and cause the amplification of noise. This type of noise is dominant in a frequency band from 10Hz up to 1kHz.

The analysis of vibrations of a structural system subjected to a given excitation becomes more difficult for increasing frequencies. The Finite Element Method (FEM) analysis of noise generated by automotive vehicles on bridges is restricted to frequencies below 100Hz due to the required elements per wavelength. For high frequencies, the Statistical Energy Analysis (SEA) is commonly used to predict noise and vibration. However the suitability of the SEA for vibration analysis becomes poorer for decreasing frequencies where modal count, SEA parameter, is not reliable.

In this research, a Z-shaped plate structure is adopted as the object of the research and estimated the sound pressure level using SEA, FEM. And an Experiment is also carried out to confirm the validity of the simulation results.

2. Simulations

Simulations are performed using SEA model and FEM model.

2.1 Statistical Energy Analysis

The concept of SEA was first introduced by R.H. Lyon and P.W. Smith as a noise and vibration estimation tool for complex structure. With the SEA method, structural components are considered as a set of equivalent vibrating elements, and evaluated the vibration condition of the element as a statistical average over the frequency band and space.

In SEA, a structure is discretised into a number of substructures called subsystems. And the response is described in terms of energy where the excitation is called input power. Lyon showed that the power flow is proportional to the difference in uncoupled energies of the resonators and that it always flows from the resonator of higher to lower resonator energy. Below, an explanation is given on basic power flow equation based on SEA.

2.1.1 Power Flow Equations

The power flow relations of a structure consisting of two subsystems are shown in Figure 1. The equations for the power flows between subsystem1 and subsystem2 under typical SEA conditions are expressed as follows.

Subsystem1: $P_{in1} = P_{diss1} + P_{trans12}$ (1.1)

Subsystem1: $P_{in2} = P_{diss2} + P_{trans21}$ (1.2)

Where intrinsic loss P_{diss1} becomes

$$P_{diss1} = \omega\eta_1 E_1 \tag{1.3}$$

In addition, transmitted power $P_{trans12}$ is expressed with the following equations.

$$\begin{aligned} P_{trans12} &= -P_{trans21} = P'_{12} - P'_{21} \\ P'_{12} &= \omega\eta_{12}E_1, P'_{21} = \omega\eta_{21}E_2 \end{aligned} \tag{1.4}$$

where η_{12} and η_{21} in SEA satisfy the reciprocity relationship $\eta_{12}n_1 = \eta_{21}n_2$. Therefore, transmitted power $P_{trans12}$ becomes

$$P_{trans12} = \omega\eta_{12}\left(n_2E_1 - n_1E_2\right) = \omega\eta_{12}n_1\left(\frac{E_1}{n_1} - \frac{E_2}{n_2}\right) \tag{1.5}$$

Consequently, power flow equations for subsystem1 and subsystem2 can be expressed as

$$\begin{aligned} P_{in1} &= \omega\eta_1E_1 + \omega\eta_{12}n_1\left(\frac{E_1}{n_1} - \frac{E_2}{n_2}\right) \\ P_{in2} &= \omega\eta_2E_2 + \omega\eta_{21}n_2\left(\frac{E_2}{n_2} - \frac{E_1}{n_1}\right) \end{aligned} \tag{1.6}$$

where n_1: modal density, η_1: intrinsic loss factor, η_{12}: coupling loss factor are called SEA parameters. If SEA parameters and input power are known, the dynamical energy distribution of the structure can be easily determined. Therefore Sound Pressure Level for each subsystem is known.

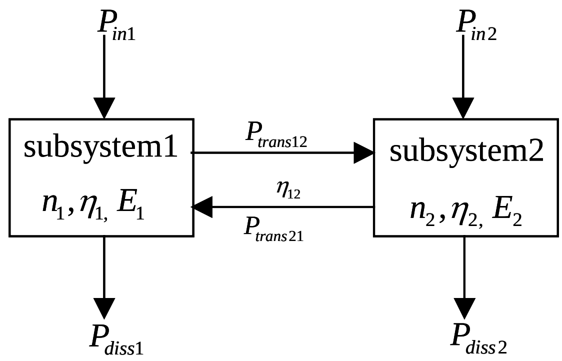

Figure 1. Power flow relation between two subsystems.

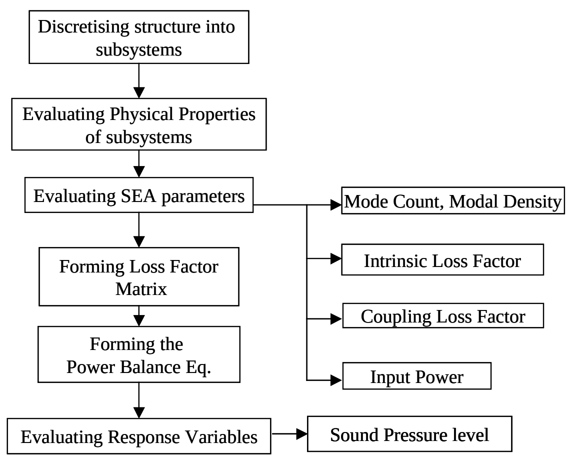

Figure 2. Calculation flow for SEA.

2.1.2 SEA Model

A Z-shaped plate structure where plates are coupled with each other at right angle is modeled for the simulations shown in Figure 3. Here, the plate structure is discretised into three subsystems and each subsystem posses the same physical and geometrical properties shown in Table-1.

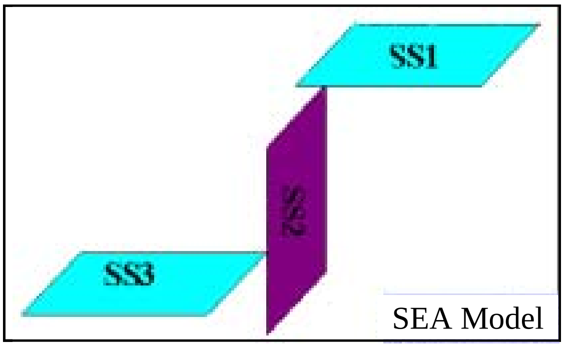

Figure 3. SEA model(Z-shaped).

Table-1 Physical properties for Subsystem1.

Physical Properties for SubSystem1	
Young modulus (N/m^2)	2.1×10^{11}
Mass density (kg/m^3)	7800
Poisson's ratio	0.3
Length (m)	0.4
Width (m)	0.2
Height (m)	0.0023

2.1.3 Estimation of SEA Parameters

Since all the subsystems of the SEA model have the same properties, only subsystem1 is discussed here.

(a) Mode Count

Mode count represents the number of resonance modes available in the band of interest for the subsystem to receive and store dynamical energy. The mode count of a structural subsystem is evaluated by using the following Equation. Figure 4 shows the mode count of subsystem1 in octave band.

$$N = \frac{S\omega}{4\pi h}\sqrt{\frac{12\rho\left(1-\nu^2\right)}{E}} \qquad (1.7)$$

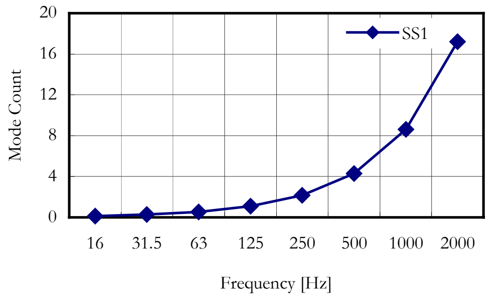

Figure 4. Mode count of structural subsystem SS1.

(b) Coupling Loss Factor(CLF)

Coupling Loss Factor gives the loss rate when power transmits from one subsystem to another. The following equation is used to evaluate the CLF of plate structure.

$$\eta_{ij} = \frac{c_g L_c \tau}{\pi\omega S_i} \qquad (1.8)$$

where c_g: the group velocity of the bending waves, L_c: the coupling length, τ :the transmission co-efficiency, S_i: the surface area, ω: the center angular frequency of the band of interest. Figure 5. shows the CLF η_{12} of the coupling between subsystem1 and subsystem2.

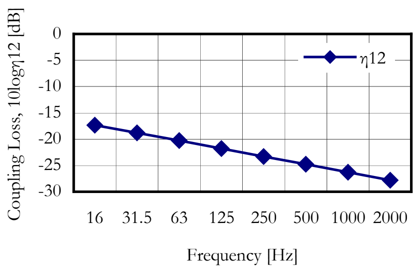

Figure 5. Coupling loss of structural subsystem SS1.

(c) Intrinsic Loss Factor(ILF)

Intrinsic Loss Factor of a subsystem represents the loss percentage when the input power to the subsystem from an external excitation source is converted to the dynamical energy of the subsystem. ILF is not possible to determine by theoretical equation like CLF. Experiment is the only way to measure it. In this research, an experimental equation shown below is used.

$$\eta_i = 0.98 \times f^{-0.7} \qquad (1.9)$$

Figure 6. shows the ILF of subsystem1.

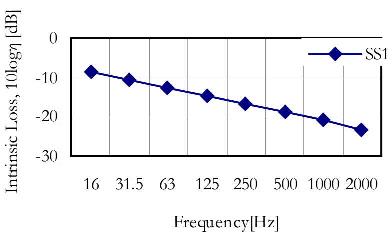

Figure 6. Intrinsic loss of Structural subsystem.

(d) Input Power (IP)

In this research, *Input Power* is evaluated by experiment. Vibration velocity is measured at 6 points on the surface of subsystem1 by exciting the structure with a vibrator. Then the input power for the structure is evaluated by using the following equation.

$$E = m\langle v^2 \rangle \qquad (1.10)$$

where m: mass, $<v^2>$: spatial square average of the vibration velocity.

2.2 Finite Element Method (FEM)

Two types of analysis, modal and harmonic, are performed by employing ANSYS/ED 5.7 FEM software. Figure 7. shows the FEM model of the plate structure which has 300 elements. Shell element with 4nodes is used. The left end of the plate structure is constrained as such that no displacements can appear here. And the physical and geometric properties of the model are same as SEA model.

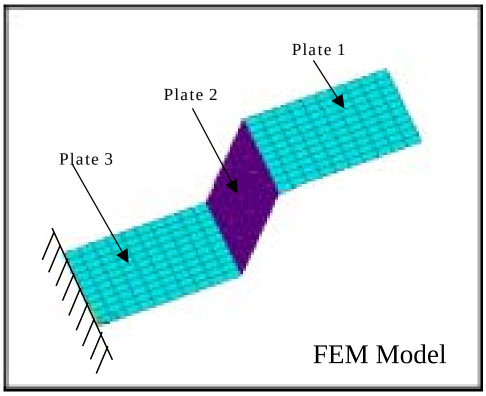

Figure 7. FEM model.

Figure 8. represents the noise analysis flow using FEM that is used in this research. And related equations are also shown here.

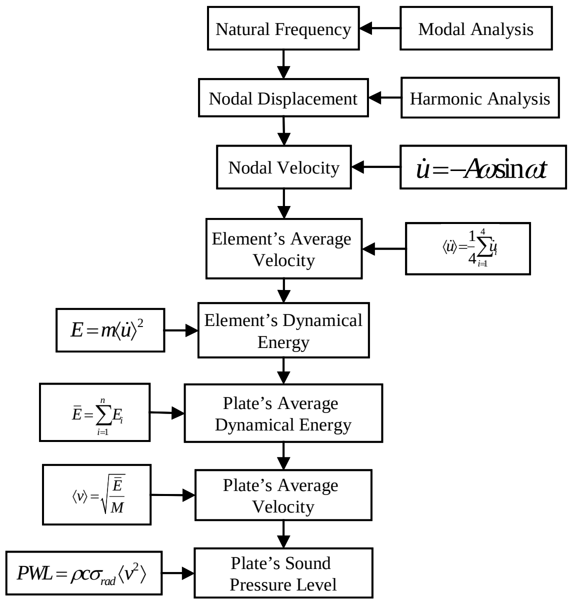

Figure 8. Noise analysis flow using FEM.

First 20 modes are calculated by modal analysis, where Block Lanczos method is used for mode expansion. Figure 9 shows the mode count that presents in each band of octave band.

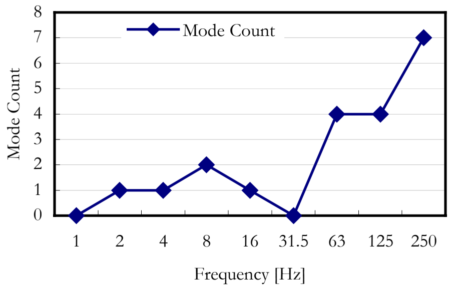

Figure 9. Mode count presents in each band of octave band.

When the natural frequencies of first 20 modes are known, a harmonic response analysis applying a sinusoidal force is performed to evaluate the nodal displacement around the resonance frequency. Maximum amplitude of the sinusoidal force is 1. And the harmonic analysis is performed from 1Hz to 177Hz.

Since the nodal displacements, calculated by harmonic analysis, are known Sound Pressure Level for each plate is determined by performing the calculations shown in Figure 8.

3. Experiment

An experiment is carried out to measure the Sound Pressure Level in order to check the validity of simulation results.

Figure 10.represents the positions of microphones and other apparatus. The left end of the plate structure is clamped that no displacements can appear. A vibrator, only excitation source for the structure, is placed at the center of the right end. Experiment is performed by changing the excitation frequency of the vibrator from 22Hz to 354Hz using an oscillator while keeping the amplitude fixed by an attenuator.

Vibration pickup accelerometer is used to measure the vibration velocity at 250Hz to evaluate the Input Power of SEA.

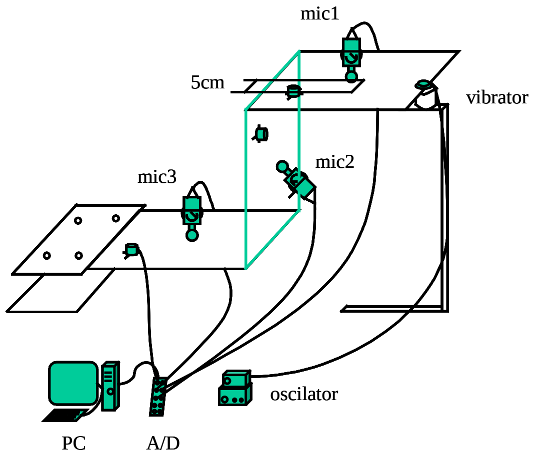

Figure 10. Positions of microphones and other apparatus

Since the microphones are set just 5cm away from each plate, no distance decay is considered.
Finally, sound pressure level for each microphone is evaluated in octave band.

4. Results and Discussions

Figure 11. shows the Sound Pressure Level in octave band evaluated by simulation, SEA and FEM, and experiment for plate1. It is known that SEA is applicable to evaluate the Sound Pressure Level at 125Hz and higher. In this research, simulated sound pressure level, SEA results, at 63Hz and above are found reliable, as the difference between simulation result and experimental result is within allowable range, 3dB. And at 31.5Hz, FEM result is closer to experimental result than SEA. Therefore FEM analysis used in this research is appropriate for 31.5Hz and less.

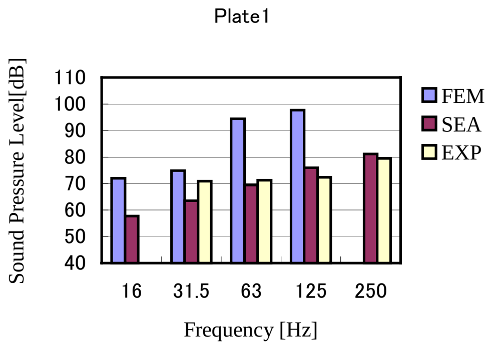

Figure 11. Comparison of results for plate1.

To be sure of the simulation results, a comparison is also performed using mode count evaluated by both SEA and FEM. It's a new approach to find the boundary between SEA and FEM. Figure 12 represents the comparison of inverse mode count.

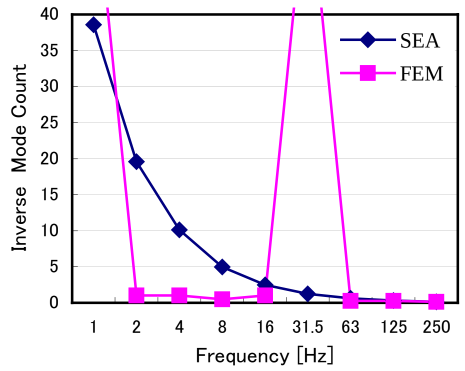

Figure 12. Comparison of inverse mode count.

According to FEM results, at 31.5Hz of octave band doesn't have any resonance mode. Therefore the 1/N line diverges at 31.5Hz. On the other hand, SEA results shows that 31.5Hz posses resonance modes. Since FEM results are reliable at low frequencies, FEM results can be considered as accurate. If the mode count is zero, the modal energy turns to infinity. Therefore the fundamental concept of SEA is collapsed.

In this research, noise analysis at 63Hz and higher frequency of octave band is possible by SEA with the required accuracy and, noise analysis at 31.5Hz and less must be analyzed by FEM where SEA is not applicable.

So, a boundary between SEA and FEM can be set considering the presence of mode count in the frequency band.

5. Conclusions

Based on the simulations and experiments conducted, the following can be concluded.

(a) Sound Pressure Level at 63Hz and higher can be predicted by SEA for this type of model.

(b) And Sound Pressure Level at 31.5Hz and below must be predicted by FEM.

(c) The highest octave band that doesn't have any resonance mode and less than that must be analyzed by FEM and rest can be analyzed by SEA with having enough accuracy.

6. Further Research

(a) Further works must be performed by changing the type of the model and the coupling patterns. Welded coupling can be raised as an example.

(b) And further experiments should be carried out to make sure that the proposed method to find the boundary between SEA and FEM works at all circumstances.

7. References

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